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COMMENTARY

From Implicit to Explicit: Articulating Equitable Learning Trajectories Based Instruction

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Over the last half century, mathematics education has seen numerous reform initiatives and standards. About every ten years, a new wave of documents offers recommendations on how to best teach mathematics. Ellis and Berry (2005) argued that although these "so-called reform" documents have increasingly attended to ideas such as mathematics for all (National Council of Teachers of Mathematics [NCTM], 1989) and equity (NCTM, 2000), they have, in fact, only generated "revisions" of mathematics instruction because they "failed to change significantly the face of the mathematically successful student" (p. 8). The Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) offers the most recent set of recommendations for mathematics reform. The document builds on the concept of learning trajectories (LT)[1] (Daro, Mosher, & Corcoran, 2011) and outlines the mathematics content and practices to be addressed at particular grade levels.

[1] While some scholars use the term *learning progression*, we use the term *learning trajectory* in this commentary to encompass both progressions and trajectories.

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With the widespread adoption of these standards, mathematics teacher educators have worked to share ideas about trajectories with teachers. Because research on learning largely developed separately from research on teaching, our work used LTs to link these two bodies of research. We theorized the concept of Learning Trajectories Based Instruction (LTBI) as a model of teaching where instructional decisions are grounded in research on student learning in the form of trajectories and we interpreted several highly developed domains of research on mathematics teaching in relation to these trajectories (Sztajn, Confrey, Wilson, & Edgington, 2012). Since that time, we have worked to share this model with teachers in professional development settings, and our research has empirically examined and elaborated the affordances of LTBI.

In our initial conceptualization, we considered LTBI to be situated within a broader landscape of equity in mathematics education because of its emphasis on the ways that instruction grounded in students' mathematical thinking provides opportunities for learning and access to rigorous mathematics instruction based on individual students' existing understandings of mathematics (Civil, 2006; Fennema & Meyer, 1989). By articulating a relation between teaching and learning, LTBI foregrounded students' thinking and suggested that students should be the primary consideration of instruction. For us, organizing instruction around these trajectories challenged the "correct/incorrect" dichotomy of ideas by acknowledging and honoring a variety of partial and alternative understandings that students have. It provided an organizational structure that aided in anticipating and explaining student learning. We proposed that LTBI pedagogical practices assisted teachers in engaging students in worthwhile tasks that engendered deep learning, eliciting and responding to kernels of important mathematical ideas, orienting students to one another's ideas in classroom discussions. These affordances advanced the work of supporting teachers in improving mathematics learning for every student.

As LTs have proliferated across educational communities, some scholars expressed concerns with issues of equity and diversity inherent in their conceptualization, development, and implementation. Some have argued that, though mathematics learning is multidimensional and occurs through connections across multiple domains, trajectories reduce learning to a hierarchical, linear path (Empson, 2011; Lesh & Yoon, 2004). Anderson and colleagues (2012) reported that researchers and other leaders in science and mathematics education have raised a number of concerns about trajectories. They pointed to theoretical framings that inadequately account for the ways culture, race, and context shape learning, challenging developers to expand the methodologies used for development and validation to ensure diverse student populations are represented in the trajectories. Another concern was related to possible translation effects as trajectories move from research to policy and practice.

Unresolved questions and concerns about the trajectories and their unintended uses led to a further examination of the LTBI model and its potential uses. In particular, we noted that our initial conceptualization explicitly attended to some aspects of equity (e.g., opportunity to learn) while leaving others tacit (e.g., race, culture, and language). In this commentary, we critically analyze the LTBI model using Gutiérrez's (2007) dimensions of equity as a comprehensive framework for equity in mathematics education. Through this theoretical examination, we make explicit the assumptions inherent in the initial model and identify opportunities for LTBI to enhance equitable mathematics instruction. First, we briefly introduce current research on LTs and highlight principles of LTs that we contend are aligned with equitable instruction. Next, we present Gutiérrez's framework and a rationale for its selection as a tool for our theoretical analysis, briefly describing each of its dimensions. We detail our analysis of LTBI and conjecture what equity-oriented uses of the model might look like in instruction. We conclude with an invitation to the mathematics teacher education community to discuss the potentials and challenges of using LTs to support equitable mathematics instruction.

Learning Trajectories

Mathematics education has increasingly attended to learning trajectories in recent years. As research-based representations of the ways students' thinking in a particular domain develops over time with instructional opportunities and supports (Clements & Sarama, 2004; Confrey, Maloney, Nguyen, Mojica, & Myers, 2009), learning trajectories are viewed by some as promising tools for aligning standards, assessments, curriculum, and instruction (Confrey, 2012; Daro et al., 2011). Research in this area initially addressed two areas: development and validation (Barrett, Clements, Klanderman, Pennisi, & Polaki, 2006; Battista, 2007; Confrey, 2012), and curriculum and assessment (Battista, 2004; Clements & Sarama, 2009). Scholars designed and empirically validated trajectories in different mathematics content areas, including whole number operations (Clements & Sarama, 2009; Confrey, 2012; Sherin & Fuson, 2005); geometry and spatial thinking (Battista, 2007; Clements & Sarama, 2009); length, area, and volume measurement (Barrett et al., 2006; Battista, 2006; Clements & Sarama, 2009); and functions (Bernbaum Wilmot, Schoenfeld, Wilson, Champney, & Zahner, 2011; Lobato, Hohensee, Rhodehamel, & Diamond, 2012). Others have sought to design curricula and assessments based on LTs (Battista, 2004; Clements & Sarama, 2007; Confrey, 2012).

More recently, LT research expanded to include a focus on instruction through examining the ways LTs might be useful in teacher education and mathematics classrooms (Edgington, 2012; Mojica, 2010; Wickstrom, 2014), while others are beginning to examine student outcomes in LT-based classrooms (Clements, Sarama, Wolfe, & Spitler, 2013; Sarama, Lange, Clements, Wolfe, & Spitler, 2012).

Evidence is accruing that outlines positive effects of LTBI, including more learner-centered classrooms rich with mathematics conversations (Clements & Sarama, 2008; Clements et al., 2013), instructional decisions based on student thinking (Bardsley, 2006; Mojica, 2010; Wickstrom, 2014; Wilson, Sztajn, Edgington, & Myers, 2015), improved understandings of student thinking (Mojica, 2010; Wickstrom, 2014; Wilson, 2009), the selection of developmentally appropriate activities (Brown, Sarama, & Clements, 2007; Edgington, 2012), and anticipations of the variety of students' conceptions (Edgington, 2012). Clements and colleagues (2013) even argued that LT-based instruction could be especially beneficial for African American students based on results of standardized measures.

Several principles unify the various conceptualizations of LTs in the field that, in our view, provide a foundation for equitable instruction. First, LTs are grounded in empirical research with students and challenge more traditional approaches to curriculum development and instruction that focus on disciplinary knowledge. In contrast with a singular development of a concept based on the logic of the mathematics, trajectories acknowledge and build from variations in students' conceptions as they engage in mathematics. Second, specific learning goals for students are clear. Though informal, partial, and alternative understandings are represented in them, LTs outline general paths that expect these earlier understandings to become more sophisticated over time. Third, trajectories are probabilistic in nature, suggesting only likely routes to learning while identifying key conceptual accomplishments along the way. This fluid nature allows for multiple points of entry for students and offers opportunities to engage ideas and coordinate them with other concepts, all the while building toward robust disciplinary understandings. Finally, LTs and student learning are necessarily dependent upon instructional opportunities. Task quality and implementation are essential in supporting student learning. This last aspect is critical and lies at the heart of LTBI—students do not simply progress along a trajectory because of maturation (Confrey et al., 2009). Learning is a product of carefully designed learning environments, well-planned learning activities, and appropriate supports from teachers (Daro et al., 2011). Thus, by developing instruction that is guided by LTs, LTBI can support more equitable instruction.

Teachers' learning about students' thinking and how it might evolve into formal mathematical concepts over time explains many of the improvements in instruction that LTBI supports. Yet the strength of using LTs in instruction—a focus on representing levels of thinking for all students—is also the cause for concerns. Such a focus precludes consideration of how students' social and cultural backgrounds shape learning and ignores the resources many students bring to instruction. Though progress along a trajectory is critical, it should not come at the expense of students' identities. Our evolving awareness of the tensions between potential benefits and consequences of different uses of LTs in teaching coupled with the critiques of LTs raised by the field led us to seek a theoretical lens to explicate the strengths of LTBI as well as illuminate opportunities for more equity-oriented uses of the model.

Gutiérrez's Dimensions of Equity

To critically examine our assumptions about equity in LTBI, we selected Gutiérrez's (2007) equity framework as a theoretical lens. Four dimensions along two axes comprise the framework. Access and Achievement are associated with the dominant axis, which represents what students need to know to participate in mainstream mathematics. In contrast, Identity and Power are dimensions of the critical axis, which represents what students need to become critical members of society. Four aspects of this conceptualization of equity supported its selection as a tool for our theoretical analysis of LTBI. First, its organization around the dominant and critical axes juxtaposed the progress toward formal mathematics content of LTs with instructional commitments to the successful development and maintenance of students' identities. Second, its dominant axis specifically attends to access, positioning it as an independent pre-cursor of achievement, which allows for specific scrutiny of the ways LBTI might be used to provide access while promoting student achievement. Third, we view the framework as inherently including elements of culturally responsive pedagogy, such as high academic achievement through cultural competence; varied instructional strategies; and links between schools, homes, and communities (Gay, 2000). Lastly, for us, the dimensions of Gutiérrez's framework provide a comprehensive representation of research on equity in mathematics education as they address both the mainstream concerns about equity (e.g., opportunity to learn, standardized test scores) as well as issues of culture, language, and socioeconomic status. Together, these aspects of the equity framework explain why we used it as a tool to elucidate strengths and identify underdeveloped areas of LTBI in relation to equity.

Access and Achievement

For a number of years, scholars in mathematics education conceived of access as opportunity-to-learn (Elmore & Furhman, 1995; Fennema & Meyer, 1989; Tate, 1995). Although opportunity-to-learn remains an important concept, it alone is insufficient in defining equity (Flores, 2007; Gutiérrez, 2007; Silver & Stein, 1996). Thus, Gutiérrez (2007) proposes that access, as one end of the dominant axis of equity, depends on resources that students physically have or do not have. It includes quality mathematics teachers, adequate technology and supplies in the classroom, a rigorous curriculum, reasonable class sizes, and supports for learning outside of class hours.

Access is a "precursor to achievement" (Gutiérrez, 2007, p. 3). Attending to access is insufficient if student outcomes are neglected. If students are provided with all of the resources mentioned above and traditional achievement patterns continue to persist, then populations of students remain underserved. For Gutiérrez, achievement is the opposite pole of the dominant axis, signaling the importance of supporting students with access in achieving. Achievement includes participation in a given class, course-taking patterns, standardized test scores, and participation in the mathematics pipeline.

Identity and Power

Gutiérrez (2007) stated:

> Because there is a danger of students having to downplay some of their personal, cultural, or linguistic capacities in order to participate in the classroom or the math pipeline … issues of identity have started to play a larger role in equity research in mathematics education. (p. 3)

Equitable mathematics instruction, therefore, must provide opportunities for students to maintain and draw upon cultural and linguistic capacity, find a balance between self and others, see themselves in the curriculum, use the curriculum as a tool to view and analyze the world, find mathematics meaningful in their lives, and sense that they have become a better person (Gutiérrez, 2007).

It is not enough to provide students with access, support achievement at high levels, and maintain students' personal identities if, "mathematics as a field and/or our relationships on this planet do not change" (Gutiérrez, 2007, p. 3). The final dimension, power, is a call for using mathematics to bring about change and social transformation. Gutiérrez suggests that this transformation may occur in a variety ways, including changes in who gets to talk in the classroom (voice), changes in who decides on curriculum, and creating opportunities for students to use mathematics to analyze and critique society.

A Critical Examination of LTBI

Our analysis of LTBI using Gutiérrez's (2007) equity framework revealed a closer alignment of the model with the dominant axis than the critical. Though the origins of LT research in assessment and curriculum development render this finding unsurprising, the analysis process highlighted assumptions implicit in our original conceptualization of LTBI as well as areas in need of greater specification. In particular, the lenses of identity and power from the critical axis highlighted areas unaddressed in the original model, identifying potential leverage points for more equity-oriented uses of LTBI. In what follows, we conjecture the ways in which LTBI might be used for equitable mathematics instruction by grounding the model in relation to the dimensions of the equity framework. Our conjectures represent both connections to empirical findings and conjectures about how LTBI might be used to assist in achieving goals of equity.

Access and Achievement through LTBI

Access requires quality mathematics instruction for each student, and we suggest that LTBI can improve this quality by: assisting teachers in attending to students' logic (Wilson, Mojica, & Confrey, 2013; Wilson, Sztajn, Confrey, & Edgington, 2014), supporting teachers in selecting or adapting rigorous and appropriate tasks within their curriculum or from other materials (Edgington, 2012; Myers, 2014; Wickstrom, 2014), and making tasks accessible for each student based on the individual conceptions of the student (Myers, 2014). LTBI supports instructional decision-making processes, including eliciting and building upon students' ideas to facilitate productive mathematical discussions that are accessible to all students, encouraging full student participation (Wilson et al., 2015).

When using LTBI, teachers may set individual learning goals for students based on their current understandings and how these understandings relate to long-term mathematical objectives (Myers, 2014). This shift from a purely disciplinary focus on mathematics to one that focuses on students' conceptions opens possibilities for teachers to meet the needs of individual students in support of their achievement, an explicit LTBI practice. Mosher (2011) stated, if children are to meet standards, "schools and teachers have to take responsibility for monitoring students' progress and intervening on a timely basis when needed" (p. 1). Further, LTBI promotes the use of cognitively demanding, open tasks to create spaces for eliciting evidence of student learning, allowing for the development of additional ways to assess what students know.

Identity and Power through LTBI

Less apparent in our analysis were direct connections to the critical axis. This finding challenged us to envision how LTBI might be used as a platform to help students see themselves in mathematics and prepare students to use their mathematical knowledge to bring about social transformation. The long-term, developmental nature of LTs assists teachers in understanding and valuing their students' mathematical ideas, thus promoting the idea that *all* students are learners and doers of mathematics. Such a view and understanding on learning promotes teachers' acknowledgment of the mathematical contributions that all students may make in the classroom and bolsters students' identities as doers of mathematics. By providing teachers with a framework for various mathematical strategies and how those strategies build toward refined mathematical concepts, LTBI sensitizes teachers to the variety of ways students solve problems and engenders respect for their approaches. This enhanced repertoire of students' conceptions encourages teachers to be open to, and accepting of, various forms of communication during mathematical discussions, connecting classroom mathematics to students' experiences outside of school.

Similar to the identity dimension, we conjectured possibilities for LTBI to scaffold power. Power in the classroom addresses the relations established among teachers, students, and society (Gutiérrez, 2007, 2009). It addresses social transformation and the ways in which mathematics can be used as a tool to critique society (Gutstein, 2003, 2006, 2007). Because LTs present a range of students' mathematical understandings over time, teachers may utilize this knowledge to ensure that students positioned along the trajectory have a voice and are provided with the opportunity to share their thinking. In contrast with a view of students as "empty vessels," LTBI assists teachers in viewing each student as knowledgeable in unique ways and in sharing that knowledge with their peers, positioning students as having expertise.

In summary, our analysis indicated that LTBI was well aligned with the dominant axis of Gutierrez's (2007) framing of equity, and research evidence is accumulating in support of these facets of the model. Our examination illuminated assumptions implicit in our original conceptualization and assisted us in strengthening the connections between LTBI and equity. Furthermore, aspects of instruction that can support students' identities as doers of mathematics and views of mathematics as a tool for social transformation unaddressed in the original model were foregrounded, leading us to develop initial conjectures about how LTBI might expand to encompass these goals. Appendix A illustrates these refined conjectures as markers of equitable LTBI in classrooms.

Discussion

Our theoretical examination of LTBI with a lens for equity resulted from a tension between concerns from the field and positive findings from research. This tension led us to question the nature of an equity-oriented use of LTBI and illuminated implicit assumptions, strengthened connections, and identified new areas where LTBI might develop in relation to equity. This process confirmed many existing findings about LTBI and its potential in classroom instruction and allowed us to situate those findings in relation to the dominant axis. This process also allowed us to bring identity and power to the foreground and envision the ways in which LTBI could be used to support the critical axis. More important, we contend that this examination and the representation of equity-oriented implementation of LTBI proposed in Appendix A can generate important discussion in mathematics education in relation to LTs.

We conclude by positing that it is not the LTBI model, but the use of the model that can be equitable or inequitable. For example, while trajectories support teachers to view student learning along a continuum, they also may allow for reifying of deficit views that justify pre-conceived ideas about "high" and "low" children, or ideas about students who do not follow the "typical" path as "deviants." Due to these potential challenges, more discussion and research are needed to understand teachers' uses of LTBI in creating equitable classrooms and challenging potential inequitable assumptions about what students can or cannot do. We continue to argue that LTBI has the *potential* to foster equity, but suggest this "potential" requires further empirical validation. Therefore, we invite our fellow researchers to engage in conversation to further explore this issue with us, to investigate the affordances and challenges of using LTs as instructional tools, and to examine the potential uses of this tool in promoting equitable mathematics instruction.

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APPENDIX A

Equitable LTBI in Classrooms

Access	Teachers engaged in LTBI ensure that students at various levels have entry points to the task.
	Teachers engaged in LTBI identify relevant and research-based materials and technology that support the development of skills represented by LTs (e.g., recognize what curricular materials are aligned with LTs and thus supportive of their learning goal).
	Teachers engaged in LTBI assess students' current mathematical understandings and determine the level of support needed to ensure students are able to access and engage with the mathematics content of the instructional task. Knowledge of LTs informs teachers' monitoring of their students' work.
	Teachers engaged in LTBI scaffold classroom discussions in ways that position all students to participate in the conversation and use knowledge of LTs to build upon students' current mathematical understanding and make connections among various mathematical ideas that arise.
	Teachers engaged in LTBI diagnose students' current understandings while focusing on future conceptions outlined by the LT to ensure that the work students are engaged in is rigorous and has the potential to help students progress toward more advanced mathematics.
	Teachers engaged in LTBI allow students to work in ways that are comfortable to them and represent their work (written work and verbal descriptions) in ways that align with the students' understanding of mathematics.
Achievement	Teachers engaged in LTBI set goals for students that are appropriate based on students' current understandings.
	Teachers engaged in LTBI distinguish what students have already learned from what they are learning and use that understanding to design instruction to advance the students' learning.
	Teachers engaged in LTBI think of a variety of ways to solicit evidence about students' understanding.
Identity	Teachers engaged in LTBI support students' efforts and encourage movement along the trajectory. Teachers use LTs to acknowledge students' current understandings as well as the knowledge that all students can progress.
	Teachers engaged in LTBI create open tasks that are relevant to and affirm their students' homes and communities.
	Teachers engaged in LTBI recognize, encourage, and determine the validity of a variety of strategies, algorithms, and tools to solve problems.
	Teachers engaged in LTBI assist students in making not only mathematical connections, but also real world connections (global, national, and local).
Power	Teachers engaged in LTBI include all students, allow all students to have voice, and ensure equitable ownership of the ideas and activities that are a part of the mathematics lesson.
	Teachers engaged in LTBI position students as experts based on their usage of certain skills or strategies.
	Teachers engaged in LTBI select or create tasks that impact the communities in which students live.
	Teachers engaged in LTBI recognize various mathematical ideas present in the classroom and encourage all students to present, justify, and defend their ideas. Teachers use LTs to facilitate discussions, orient students to other, and make mathematical connections.
	Teachers engaged in LTBI frame *every* student as a creator of mathematical knowledge, recognize what students already know, and *self*-empower students by helping them see themselves as doers of mathematics.